

# INDEX

- Absolute convergence, 591; *see also* Maxima and minima of functions
- Absolute value, of complex number, 35  
function, 24 (Problem 9)  
of real number, 4  
rules for, 4
- Acceleration vector, 212  
in polar coordinates, 437-443  
tangential and normal components of, 213, 428
- Addition, of ordinates, 90  
of vectors, 49
- Algebraic, equations, 38, 444-447  
function, 383
- Alternating series, 595
- Analytic functions, 384, 618
- Angle, function, 359-364, 422  
in polar coordinates, 35  
between radial and tangent vectors, 435  
in trigonometry, 31-38  
between two curves, 196  
between two lines, 79-80  
between vectors, 64, 71
- Angular velocity, 422, 438  
acceleration, 438
- Antiderivative, 238
- Approximately equal, 230-231
- Arc length, 329-338, 341-343  
differential of, 192, 336  
derivative of, 333  
function, 333  
as parameter, 335  
of piecewise smooth path, 341  
of smooth path, 329
- Archimedes' method of exhaustion, 301
- Archimedean order, 131  
(Problem 7)
- Area, 245-249, 299-304  
under a curve, 301-307  
enclosed by closed path, 474, 478  
general formula for, 476-482  
of line, curve, 301  
by method of exhaustion, 301  
of parallelogram, 73  
in polar coordinates, 473  
of surface of revolution, 501  
of surface of sphere, 501  
between two curves, 471  
vector formula, 481
- Arithmetic progression, 42
- Associative law, 3, 50
- Asymptote, horizontal, 88-89, 118, 405  
for hyperbola, 29  
vertical, 88, 89, 405
- Asymptotic curve, 405
- Average value of function, 318
- Axes, of conic sections, 26, 28-30  
orientation of, 416  
rotation of, 415  
translation of, 414
- Basis, 59, 116, 271, 385  
(Problem 2(i)), 544  
orthogonal, 65  
orthonormal, 65  
theorem, 59
- Best circular approximation, 429
- Best linear approximation, 163  
(Problem 21)
- Binomial, coefficients, 44, 125  
series, 620  
theorem, 620
- Boundedness, 95 (Problem 18), 127, 129, 569
- Boyle's law, 177 (Problem 4)
- Cardioid, 474
- Cartesian coordinate system, 9
- Cauchy's condition, for sequence, 571  
for series, 583
- Cauchy's Mean Value theorem, 469
- Cavalieri's principle, 498
- Center of mass, 77 (Problem 9), 505-520
- Centripetal force, 438
- Centroid of region, 514-520
- Chain rule, for derivatives, 163, 165, 192, 229  
for differentials, 229
- Change of coordinates, effect on orientation, 416  
by rotation, 415  
by translation, 414
- Circle, 27  
general equation for, 27  
of infinite radius, 425  
point, 28
- Coefficient of expansion, 140
- Combination, 44
- Commutative law, 3, 50
- Comparison tests for series, 584-586
- Complex numbers, 35  
absolute value or modulus of, 35  
argument or amplitude of, 36  
conjugate of, 35  
imaginary part of, 36  
real part of, 36
- Complex power series, 630
- Complex-valued function, 376
- Component, of a vector, 62, 65  
in direction of a vector, 67  
radial and transverse, 433  
tangential and normal, 428
- Composite function, 85  
domain of, 86
- Compound interest, 152  
(Problem 6)
- Concave, down, 202, 402  
function, 402  
upwards, 202, 402
- Conditionally convergent, 591
- Conduction of heat, 523
- Cone, surface area of, 500  
volume of frustum of, 495  
(Problem 11)
- Congruent figures, 300
- Conic sections, 30
- Conservation of energy, 522
- Continuity, 87, 101-114, 188, 376  
of a difference, 107  
of composite function, 109  
of exponential function, 111, 371  
of inverse function, 109  
of logarithmic function, 111, 371, 373  
of polynomial, 111  
of product, 107  
of quotient, 107  
of rational function, 111  
of sum, 107  
of sum of a power series, 609

- Continuity (cont'd)  
of trigonometric functions, 111, 355  
of vector function, 188
- Continuous function, 87, 101-114, 188, 376  
definite integral of, 288  
indefinite integral of, 243  
properties of, 105-110, 132-138  
Riemann integral of, 323-328
- Continuous indefinite integral of piecewise continuous function, 281
- Convergence, absolute, 591  
conditional, 591  
pointwise, 603  
of sequence, 127, 568, 571-572  
of series, 575
- Convex function, 402
- Corner, 88, 89, 391
- Critical point, 386
- Curvature, 424-432, 440  
center of, 430  
circle of, 425, 429, 430 (Problem 4)  
formulas for, 425  
in polar coordinates, 440  
radius of, 424
- Curve, closed, 474  
in polar coordinates, 432
- Cusp, 88, 89, 405
- Cycloid, 186 (Problem 3)
- Definite integral, 240-244, 285-349, 470-565  
as an accumulator, 307-310  
as a greatest lower bound, 288  
as a least upper bound, 296  
as a limit, 322  
as a linear mapping, 327  
of continuous function, 288  
geometric meaning of, 245-249, 289  
inequalities for, 317  
integration by parts, 311, 315-316 (Problem 6)  
of linear combination, 295  
lower limit of, 288  
mean value theorem for, 318  
numerical evaluation of, 544-565  
of piecewise continuous function, 338  
properties of, 291-298  
of speed, 332  
substitution in, 312  
as total displacement, 346  
upper limit of, 288
- DeMoivre's formula, 36
- Derivative, as a mapping, 237  
of arc length, 333  
of  $a^x$ , 173-174, 371-372  
of composite function, 163, 165, 192, 208-210, 229  
computation of, 145  
definition of, 144  
of  $e^x$ , 153, 173, 371-373  
not existing, 148  
geometric significance of  
higher, 200  
of hyperbolic functions, 161-162 (Problem 12), 379  
of implicit functions, 177-183  
of inverse function, 169-175  
of inverse trigonometric functions, 173  
of  $\log_a x$ ,  $\ln x$ , 158-159, 370-373  
mean value theorem for, 220  
notation for, 148  
order of, 200  
of product, 153, 192  
of quotient, 153  
of related functions, 175  
rules for calculating, 153-160  
of second or higher order, 200-214  
of sum, 153  
as slope of tangent, 145-147  
of trigonometric functions, 153, 157, 350-359  
of vector functions, 189-193  
of  $x^n$ , 153, 156  
of  $x^a$ , 167, 172
- Determinant, 16-19, 73  
as a polynomial, 18  
expansion by minors, 17  
minor of a, 17  
order of a, 16, 17  
use in solving linear equations, 18
- Differentiable function, 149
- Differential, 225-235  
and approximation, 230-232  
of arc length, 336, 434  
chain rule for, 229  
rules for calculating, 228  
and tangent, 232
- Differential equation, 213-214 (Problems 3, 4), 224-225 (Problems 11-13), 263-264 (Problems 68-72), 536-554, 627-630  
characteristic equation for, 546  
degree of, 537  
existence theorem for, 538  
first order, 539-541  
homogeneous, first order, 540  
homogeneous linear, 545  
initial conditions or values for, 538  
linear of first order, 541  
linear of order  $n$ , 543  
linear of order 2, 543-554  
order of, 537  
ordinary, 536  
power series solution of, 627-630  
separation of variables in, 263, 539  
solution of, 537  
variation of parameters, 548
- Differentiation, chain rule for, 163  
implicit, 179  
logarithmic, 181-182  
rules for, 153-175  
of vector functions, 192  
*see also* Derivative
- Directed angle, 72
- Directed line segment, 46-47
- Directrix, 26, 28-31
- Discontinuity, 88-89, 101, 404
- Displacement, 48
- Distribution, of electric charge, 509  
of energy, 509
- Distribution of mass, 504-520  
continuous, 506  
on a curve, 515-517  
discrete, 505  
homogeneous, 512  
in a plane, 510  
in space, 519-520 (Problems 17-22)
- Distributive law, 3, 53, 70
- Divergence, proper or oscillatory, 568, 575;  
*see also* Convergence
- Domain of a function, 19-20
- Dominant series, 584
- Dummy variable, 289
- $e$ , 159-160, 373-374
- Eccentricity of conic section, 28-31
- Elimination of  $xy$  term, 420
- Ellipse, 28, 398, 414-417
- Ellipsoid, 495 (Problem 13)
- Elliptic integral, 359 (Problem 8)
- Estimation of error, 448-453  
linear estimates, 448-449  
in Newton's method, 457-459  
quadratic estimates, 449-450
- Equation, algebraic, 38  
differential, *see* Differential equation  
equivalent, 11  
graph of, 10  
homogeneous linear, 15-16, 19  
linear, 11-19  
of second degree, 416
- Euclidean coordinate transformation, 417
- Euler identity, 377
- Even function, 313
- Evolute of path, 430
- Existence theorem for differential equations, 538
- Exp function, 370
- Exponential decay, 552

- Exponential function, 40, 370,  
 Appendix III  
 derivative of, 153, 173-174,  
 371-372  
 integral of, 250  
 properties of, 40, 371  
 Exponents, 39  
 Extension, of a function, 355  
 rules, 355
- Factorial, 43, 126  
 Falling body, 150 (Problem 8)  
 Fluid pressure, 522  
 Focus of conic section, 25, 28-31  
 Forces, 75, 309, 439, 521-523  
 in equilibrium, 75  
 resultant of, 75  
 Fourier, coefficients, 635  
 cosine series, 639 (Problem 4)  
 series, 635-636  
 sine series, 639 (Problem 5)  
 Frustum of cone, 495 (Problem  
 11), 500
- Functions, 19, 84  
 algebraic, 383  
 analytic, 384, 618  
 angle, 359-364, 421-422  
 arithmetic mean of, 319  
 average value of, 318-319  
 classification of, 383-384  
 complex exponential, 377  
 complex-valued, 376  
 composite, 85  
 continuous, 87, 101-114, 188,  
 376  
 definite integral of, see Definite  
 integral  
 derivative of, see Derivative  
 differentiable, 149  
 domain of, 19-20  
 equality of, 20  
 even, 313  
 exponential, 40, 368-374  
 extension of, 355  
 graph of, 22, 87  
 hyperbolic, 378-380  
 identity, 21, 87 (Problem 8)  
 implicit, 178  
 indefinite integral of, see  
 Indefinite integral  
 inverse, 21, 91-95, 169-175  
 linear, 22, 23  
 logarithmic, 40-41, 368-374  
 maximum or minimum of, 87-  
 89, 110, 214-218, 386-400,  
 404-413  
 multiplicity of zero of, 22  
 numerical evaluation by  
 power series, 623-625  
 odd, 313  
 piecewise continuous, 138  
 (Problem 12), 280-281, 338  
 polynomial, 22, 23, 383  
 range of, 19-20  
 rational, 22, 23, 267-277,  
 383  
 of real variable, 21-22  
 Riemann integral of, 323  
 root mean square of, 319  
 of several real variables, 23  
 step, 281  
 transcendental, 383-384  
 vector, 84, 187  
 zero of, 22
- Fundamental theorem, of  
 algebra, 38  
 of calculus, 294
- Geometric, invariant, 417  
 optics, 198 (Problem 3)  
 progression, 45 (Problem 2)  
 series, 578  
 Geometry of plane curve, 199  
 (Problem 5)  
 Gradient, of a road, 141, 149  
 (Problem 2)  
 temperature, 140, 152  
 (Problem 10)
- Graph, of equation, 10  
 of exponential function, 40  
 of  $f + g$ ,  $f - g$ ,  $fg$ , 90-91  
 of function, 22, 87, 403-421,  
 432-443, 448-453  
 of inequality, 10  
 intercepts of, 11  
 of logarithmic function, 158  
 of second degree equation,  
 24-31, 414-421  
 of trigonometric functions,  
 34
- Graphing techniques, see Graph  
 of function
- Harmonic, motion, 551  
 series, 576  
 Higher derivatives, 200-214  
 of composite function, 208  
 of implicit function, 210  
 of vector function, 211-213  
 Hooke's law, 527 (Problem 7),  
 550
- Horizontal inflection point,  
 216, 387  
 Hyperbola, 29-30  
 Hyperbolic functions, 161-162  
 (Problem 12), 378-381  
 relation with trigonometric  
 functions, 380  
 Hyperboloid, 495 (Problem  
 14)
- Implicit differentiation, 179  
 function, 178-183  
 Improper integral, 528-536  
 comparison rule for, 535  
 convergent, 529  
 divergent, 529  
 principal value for, 536  
 properties of, 534  
 Impulse, 521  
 Indefinite integral, 236-285,  
 365-367  
 approximate methods, 281-284  
 of complex function, 376  
 of continuous function, 243  
 of functions given by different  
 formulas, 278-281  
 by graphical methods, 247  
 integration by parts, 265  
 partial fractions and, 267-278  
 of piecewise continuous func-  
 tions, 281, 284  
 of rational function, 267-278  
 of rational function of  $\sin x$ ,  
 $\cos x$ , 365  
 reduction formulas for, 266  
 (Problem 16), 367-368  
 (Problems 3 and 4)  
 rules for, 250, Appendix I  
 of step function, 281  
 substitution rules, 254-261  
 of sum, 251  
 of vector function, 347-349  
 Indeterminate forms, 122, 460-  
 469  
 evaluation by L'Hospital's  
 rules, 460-464  
 evaluation by Taylor's formula,  
 460
- Induction, definition by, 43  
 mathematical, 41  
 principle of, 41  
 Inequality, 3-4  
 graph of, 10  
 for integrals, 317-320  
 rules for, 3-4
- Infinite decimal, 2, 566-567  
 Infinite sequence, 125, 568  
 bounded, 127, 568  
 convergent, 127, 568  
 divergent, 127, 568  
 monotone, 127, 568  
 null, 568  
 oscillatory divergent, 568  
 of partial sums, 575  
 properly divergent, 568  
 subsequence, 569-570  
 Infinite series, 566-640  
 absolute convergence of, 591  
 Cauchy product, 600  
 convergent, 575  
 divergent, 575  
 geometric, 578  
 harmonic, 576  
 partial sum of, 575  
 products of, 598-601  
 properties of, 580-583  
 rearrangement of, 597  
 sum of, 575

- Infinite series of functions, 602-640
  - M-test for convergence, 604
  - pointwise convergence, 602-603
  - region of convergence, 602-603
- Inflection point, 202, 216, 218, 387, 402
- Initial conditions for differential equation, 538
- Inner product, 67-71
  - properties of, 68-70
  - rules for, 70
- Instantaneous, acceleration, 143
  - rate of change, 143
  - velocity, 142
- Integral, definite, *see* Definite integral
  - improper, *see* Improper integral
  - indefinite, *see* Indefinite integral
  - line, *see* Line integral
  - as sum of signed areas, 470
  - test for series, 587
- Integrand, 238
- Integrating factor, 541
- Integration by parts, 265, 311, 315-316 (Problems 6 and 7)
- Intercepts of a graph, 11
- Intermediate value theorem, 108-109, 135
  - for derivatives, 235 (Problem 13)
- Intermolecular potential, 389
- Interpolation, 232, 234
- Intersection of sets, 6
- Interval, 6-7
- Invariant, 417
- Inverse function, 21, 91
  - derivative of, 169-175
  - domain of, 91
- Inverse trigonometric functions, 91-95, 173
- Irrational numbers, 1-2
- Jump discontinuity, 88, 89
- $K$  (curvature), *see* Curvature
- Kepler's equation, 225
- Kepler's laws, 225, 438, 438-440, 442 (Problem 4), 486 (Problem 7)
- Kinetic energy, 521-522
  - of continuous medium, 523
- Lagrange multiplier, 398
- Law, of conservation of energy, 522
  - of cosines, 359 (Problem 7)
  - of gravitation, 205, 392 (Problem 4), 439
  - Kepler's, *see* Kepler's Laws
  - of natural growth, 207 (Problem 13)
  - Newton's, 205, 537
  - of reflection, 390
- Least squares, 395 (Problem 20)
- Least upper bound, 129, 135
  - axiom, 129
- Left turn operation, 71
- Leibnitz's rule, 200
- Lemniscate, 475 (Problem 2)
- Length of path, 331 (*see also* Arc length)
- L'Hospital's Rules, 460-467
  - proof of, 464-467
- Limacon, 475 (Problem 2)
- Limit, of composite function, 108, 133
  - of continuous function, 112
  - of difference, product, quotient or sum, 105, 133-135, 188
  - of a function, 88-89, 95-138, 187-188
  - infinite, 120
  - to the left, right, 97
  - of a sequence, 126, 568
  - uniqueness of, 100, 104 (Problem 8)
  - as  $x \rightarrow \infty$  or  $-\infty$ , 118
- Linear differential equations, 541-554
- Linear equations, 11-14, 78-81
  - simultaneous, 14-19
- Linear independence and dependence, 58, 115
  - criterion for, 58
  - see also* Basis
- Linear interpolation, 232, 234 (Problem 8)
- Linearly independent set of functions, 115, 208 (Problems 15, 17), 544
- Linearization, 233
- Linear mapping, bounded, 327
  - definite integral as, 327
  - derivative as, 237
- Line equation, 12-13, 78-81
  - intercept form, 13
  - normal form, 82 (Problem 14)
  - parametric form, 80
  - point-slope form, 13
  - two-point form, 13
  - vector form, 78
- Line integral, 478-487, 493-496, 513
- Lines, 12-14, 78-83
  - angle between two, 79
  - angle of inclination of, 13
  - normal vector to, 80
  - parallel, 13
  - perpendicular, 13
  - slope of, 12
  - tangent, 141, 146, 192, 197, 232
- Load on a beam, 527
- Logarithm, 41, 155, 158-160, 370-374
  - derivative of, 158
  - Napierian or natural, 41, 370
- Logarithmic, differentiation, 181
  - function, 41, 370-374, Appendix IV
- Lower bound, 130
  - greatest, 130
- Lower sum, 288
- Maclaurin series, 618
- Mapping, 21
  - into and onto, 21
  - one-to-one, 21
- Marginal analysis, 140
- Mathematical induction, 41
- Maxima and minima of functions, 87, 89, 110, 136, 214-218, 386-400
  - with side condition, 395-400
- Mean Value theorem, for definite integrals, 318
  - for derivatives, 220
- Mesh of a subdivision, 321
- Method of least squares, 395
- Moment, of force, 505
  - of inertia, 506, 511
  - $k$ th, 505, 507, 510, 516
- Momentum, 521
- Monotone, functions, 88, 91, 172, 221, 224
  - sequences, 127, 568
- Multiplication of scalars, 52-55, 114-115
- Newton's law of gravitation, 205
- Newton's method, 444, 446-447, 457-459
  - error in, 457-459
- Newton's second law, 205, 537
- Normal line to curve, 196
- Null sequence, 568
- Number, axis, 1
  - system, 3
- Numerical evaluation of function
  - by power series, 623-625
- Numerical evaluation of integrals, 247, 554-564
  - by Simpson's rule, 558-561, 562-564
  - by trapezoidal rule, 555-557, 561-562
- Odd function, 313
- One-to-one correspondence, 21
  - between vectors and points, 48
- Orientation of axes, 416
- Orientation of plane, 73-74, 416
  - oppositely oriented bases, 73
  - similarly oriented bases, 73
- Orthogonal vectors, 64
- Oscillatory discontinuity, 88, 89
- Pappus, theorems of, 515, 517

- Parabola, 25-27  
   reflection property of, 198  
   (Problem 3)  
 Parallel axis theorem, 514  
 Parallelogram law, 49  
 Parameter, 80, 183  
   change of, 334  
 Parametric equations 80, 183-186  
 Partial fraction expansion, 267-268, 270-274  
 Paths, 183  
   angle between, 196  
   angle function for, 359-364  
   area enclosed by closed, 474, 478-484  
   closed, 480  
   equivalent, 334  
   geometry of, 199 (Problem 5)  
   normal line to, 196  
   piecewise smooth, 480  
   simple closed, 480  
   smooth, 480  
   tangent line to, 196, 197  
   vector equation, 197  
 Pendulum, 554 (Problem 5)  
 Percentage error, 231  
 Periodic functions, 404, 634  
 Periodicity, 404  
 Permutation, 44  
 $\pi$  (the number), 352  
 Piecewise continuous function, 138, 281  
 Pointwise convergence, 603  
 Polar coordinates, 33  
   acceleration vector in, 437  
   angle between radial and tangential vectors, 435  
   arc length in, 434  
   area in, 473  
   argument of, 35  
   curvature in, 440  
   curves in, 432-442  
   modulus of point in, 35  
   polar angle in, 35  
   polar distance in, 35  
   radial vector in, 433  
   tangential vector in, 433, 435, 442 (Problem 7)  
   transverse vector in, 433  
   velocity vector in, 433  
 Potential energy, 392, 522  
   of molecular forces, 527 (Problem 8)  
   of stretched spring, 175  
 Power series, 604-633  
   center of, 604  
   coefficients of, 604  
   complex, 630-633  
   differentiation of, 609, 612-614  
   integration of, 609, 612-614  
   inverse of, 616 (Problem 9)  
   properties of, 608-615  
   radius of convergence, 605  
     reciprocal of, 615 (Problem 7)  
 Primitive of a function, 238  
 Principal value of inverse sine, 93  
 Probability, 524-526  
   density function, 525  
   distribution function, 525  
   normal density, 525  
 Product, Cauchy, 600  
   of series, 598-601  
   *see also* Inner product  
 Properties of real number system, 3  
 Pythagorean theorem, 8  
  
 Quadratic formula, 22  
  
 Radial vector, 433  
   component, 433  
 Radian, 31  
   measure, 32  
 Radius, of convergence, 605-607  
   of curvature, 213, 424  
   of gyration, 506  
 Range, of a continuous function, 110  
   of function, 19  
 Ratio test, 592, 594 (Problem 18)  
 Rational numbers, 1  
 Real number, axis, 1  
   system, 1-3  
 Rearrangement of series, 597  
 Reflection of light ray, 390  
 Region, centroid of, 514  
   of convergence, 603  
   doubly connected, 484-485  
   simply connected, 484-485  
 Resonance, 553  
 $\rho$  (radius of curvature), 213, 424  
 Riemann integral, 323-327, 339-340, 344-345  
   of continuous function, 325  
   of piecewise continuous function, 339-340  
   of vector function, 344-345  
 Rolle's theorem, 219  
 Root test, 593, 594 (Problem 19)  
 Root mean square of a function, 319  
 Rotation of axes, 415  
  
 Scalars, 49, 115  
 Sensitivity, 139  
 Separation of variables, 263 (Problem 68), 539-540  
 Sequences, 125-127, 566-640  
   bounded, 127  
   convergent, 127  
   divergent, 127  
   finite, 125  
   of functions, 602  
   infinite, 125  
   limit of, 126, 568  
   monotone, 127  
   of partial sums, 575  
   theorems on, 127, 129, 131, 568-572  
 Set, 2, 5-6  
   element of a, 6  
   empty, 2, 6  
   finite, 2  
   infinite, 2  
   subset of, 6  
 Sets, intersection of, 6  
   union of, 6  
 Simple harmonic motion, 551-552  
 Simpson's rule, 558-561, 562-564  
   error in, 561  
 Slope of tangent to curve, 141  
 Solid of revolution, 487  
   centroid of, 519 (Problem 17)  
   ellipsoid, 495 (Problem 13), 515  
   hyperboloid, 495 (Problem 14)  
   moment of inertia of, 519 (Problem 20)  
   volume by cylindrical shells, 489-491  
   volume for parametric formula, 493-495  
   volume for polar coordinates, 492  
   volume by slicing, 488  
 Solution of differential equation, 537  
   general or particular, 537  
   as a power series, 627-630  
 Speed, 99, 191, 213  
   instantaneous, 99, 191  
 Sphere, 8, 11  
   area of surface, 8, 501  
   volume of, 8, 409  
 Square root algorithm, 445  
 Square wave, 637  
 Step function, 281  
 Subdivision of interval, 241  
   mesh of, 321  
 Subnormal to curve, 199 (Problem 5)  
 Subsequence, 569-571  
 Subset, 6  
 Subtangent to curve, 199 (Problem 5)  
 Surface area, 500  
   of cone, 9, 500  
   of cylinder, 9, 500  
   of frustum of cone, 500  
   of sphere, 501  
   of surface of revolution, 501  
 Surface of revolution, 501  
 Symmetry, 404  
  
 Tangent line, 141, 146, 192, 197, 232, 442 (Problem 7)  
   using differentials, 232  
 Taylor's formula with remainder in integral form, 616  
   in Lagrange's form, 453-456, 616-618

- Taylor series, 618-627
- Tests for convergence of series,
  - Cauchy condition, 583
  - comparison, 584-586
  - integral, 587-589
  - ratio, 592, 594 (Problem 18)
  - root, 593, 594 (Problem 19)
- Tetrahedron, volume of, 497
- Theorems of Pappus, 515, 517
- Torsional vibrations, 554 (Problem 6)
- Torus, 491
  - surface area of, 503
  - volume of, 491, 494
- Total displacement, 346
- Transcendental function, 383
  - elementary, 383
- Transients, 553
- Translation of axes, 414
- Transverse vector, 433
  - component, 433
- Trapezoidal rules, 555-557, 561-562
  - error in, 557
- Travel economy, 393
- Triangle inequality, 8
  - for vectors, 50
- Triangular wave, 637
- Trigonometric functions, 32, 350-359
  - derivatives of, 157-158, 353, 356
  - graphs of, 34
  - identities, 357, Appendix V
  - inverse of, 173
  - tables of, Appendix II
- Trigonometric series, 633
- Union of sets, 6
- Unit vector, 48
- Upper bound, 129
  - least, 129, 135
- Upper sum, 288
- Variable, independent or dependent, 20
- Variation of parameters, 548
- Vector function, 187
  - component functions of, 188
  - continuity of, 188
  - definite integral of, 344
  - derivative of, 189
  - indefinite integral of, 346-347
  - limit for, 188
  - Reimann integral of, 344-345
- Vectors (in the plane), addition
  - of, 49-50
  - angle between, 64
  - associative law, 50
  - collinear, 49
  - commutative law, 50
  - components of, 62
  - distributive law, 54
  - dot product, 67
  - functions, 187
  - inner product, 67
  - linear combination of, 59
  - linearly dependent, 58
  - linearly independent, 58
  - multiplication by scalars, 52-54
  - orthogonal, 64
  - parallel, 49
  - perpendicular, 64
  - subtraction of, 51
  - sum of, 49
  - triangle inequality for, 50
  - $x$ -component,  $y$ -component, 65
- Vector space of functions, 115, 177 (Problems 5, 6, 7), 208 (Problems 15, 16, 17), 225 (Problems 16, 17), 385 (Problem 2)
  - basis for, 116, 208 (Problem 16)
  - dimension of, 117
- Velocity of a particle, 141
- Velocity vector, 190, 211
  - invariance of, 192
- Vertical tangents, 391
- Vibrations, 550-554
  - critically damped, 552
  - damped, 552
  - resonance, 553
  - sympathetic, 553
  - torsional, 554 (Problem 6)
  - transients, 553
- Volume, of cone, cylinder, parallelepiped, prism, pyramid, sphere, 8-9
  - of solid of revolution, 487-496
- Wave, square, 638
  - triangular, 637
- Weierstrass M-test, 604
- Winding number, 363
- Work, 68, 309-310, 520
- Wronskian determinant, 550 (Problem 9)
- Zero, function, 94 (Problem 9)
  - of a function, 22